BROAD BAND CRYSTAL WAVE FILTERS

BY

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PREFACE

The intention of this paper is to correlate the small amount of material available on the subject of broad band crystal wave filters and to describe the procedure and problems involved in the construction of the filters.

Acknowledgment is made to the Bendix Radio Division of the Bendix Aviation Corporation for the use of their laboratories and to Mr. Paul D. Rockwell of Bendix Radio for his advice, encouragement and aid in making this project successful.

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LIST OF SYMBOLS

C	capacity, compliance
C_A	lumped crystal shunting capacitance
C_m	motional compliance
C_o	crystal plating capacitance
C_{12}	capacity between crystal plates 1 and 2
db	decibel
f	frequency in cycles per second
f_A	anti-resonant frequency
f_R	resonant frequency
f_{1_ℓ}	frequency where lowest cut-off occurs
f_3	frequency where highest cut-off occurs
f_{1_ℓ}	first critical frequency in lattice arm
f_{1_s}	first critical frequency in series arm
IF	intermediate frequency
j	complex operator $\sqrt{-1}$
L	inductance
l_y, l_w, l_t	length, width, thickness of crystal
m	network parameter
Q	quality factor, ratio of reactance to resistance
Q_1, Q_2	symbol for crystals 1 and 2
R	resistance
R_m	motional resistance
X	reactance
Z	impedance
Z_1	series impedance
Z_2	shunt impedance

Z_o	iterative or characteristic impedance
Z_{oc}	open circuit impedance
Z_{sc}	short circuit impedance
α	attenuation factor
β	phase shift factor
γ	propagation constant
ω	angular frequency - $2\pi f$
ω_n	$2\pi f_n$
ϕ	transformation ratio

CHAPTER I

INTRODUCTION

The interest for crystal wave filters that prompted the selection of this subject for a thesis came from the assignment of the task to design, build and test a broad band crystal filter for Bendix Radio for a sharply selective intermediate frequency filter. This filter was to provide a ± 2.5 kilocycle bandwidth at 135 kilocycles with a maximum attenuation of 6 d.b. down and, along with its associated circuits, was to have a 60 d.b. attenuation at ± 5 kilocycles from the mean frequency. In order to accomplish this task extensive reading was necessary, as there is a relatively small amount of literature on the subject. Many questions were left unanswered by the study so that it is felt that the experience gained from the design and construction of this filter should be transcribed to aid any engineer who is confronted with a similar problem in the future.

The original design work on crystal wave filters was done by W. P. Mason for the Bell Telephone Laboratories. The reason for the development was to provide selective frequency bands on a multi-channel carrier telephone system. Conventional filters had been used but due to the inherent losses of inductors, cut-off frequencies as sharp as desired were not obtainable. This led to the consideration of substituting crystals for the lumped circuit elements, a concept originally suggested by L. Espenschied of the

American Telephone and Telegraph Company.

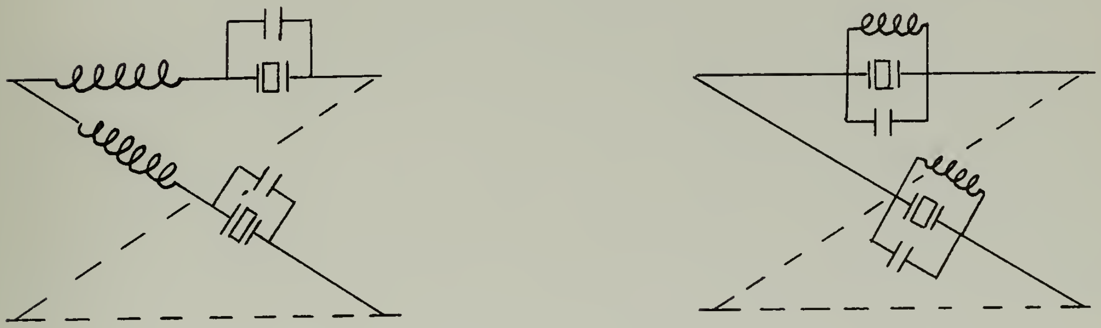
A quartz crystal is a mechanical vibrational element whose mass is very large compared to its damping resistance. From the study of this vibrating plate it was found that its vibrational resonance frequency was a function of its dimensions. Its piezo-electric properties permit its use in electrical circuits therefore it appeared to be a practical substitute for condensers and inductors. The circuitry and further discussions of crystals in this application in filters is included in a later portion of this report.

Espenschied's work on crystals employed as filter elements was done with ladder type filters. He found that the bandwidth obtainable by ladder type circuitry was too narrow for telecommunications, being of the order of 480 cycles for a 60 KC carrier.

Mason studied the application of crystals to the lattice type filter, developed by Dr. G. A. Campbell of American Telephone and Telegraph and found that by the addition of inductance to the circuit, the ratio of the upper to lower cut-off frequency was increased to 1.135 as compared to 1.008 of the ladder filter. This made the crystal filter a practical device for a broad band pass filter for use in carrier telephony.

The band pass lattice type crystal wave filter can take two forms depending upon the iterative impedance desired of the filter. In Fig. 1.1a is shown a broad band crystal wave filter that has a 600 ohms iterative impedance

at 60 kilocycles which reduces to 25 ohms at 500 kilocycles. Fig. 1.1b is a sketch of a high impedance filter that will have an iterative impendance of approximately 400,000 ohms at 500 kilocycles.



(a) (b)

Fig. 1.1 Broad Band Crystal Lattice Filters

The low impedance filter lends itself to application in telephone transmission systems where matching of cables is important. The high impedance filter may be used in intermediate frequency sections of radio receivers where it is necessary to operate into a high impedance load.

With the coming of subminiturization in electronic equipment the crystal filter represents a likely prospect as an I.F. filter for maximum selectivity in a minimum space, especially in low frequency application. The crystals are necessarily small and by combination of components these filters can take a small dimension, of the order of 3/4"x1"x1½". By the filter's inherent sharp selectivity the remainder of the I.F. stages can be simple single

tuned circuits thus reducing their size and tuning difficulties. Several manufacturers of electronic equipment are working on this problem at the present time and the work described in Chapter VI is only one approach to the problem.

CHAPTER II

BASIC THEORY OF BAND PASS FILTERS

To cover the complete theory of filters, or even band pass filters, would require more space than this entire thesis, therefore, only the basic concepts of filters will be touched with lattice type filters stressed.

Filters are specialized networks that discriminate between frequency bands in their ability to pass or reject particular bands. An ideal filter would give no attenuation to the desired frequencies and infinite attenuation to the undesired frequencies but since the components of filters are physically realizable inductors and capacitors a certain amount of loss is inherent, therefore the ideal can only be approached.

From long line theory the equation for the iterative impedance of a T network is:

$$Z_o = \sqrt{Z_{oc} Z_{sc}} \qquad 2.1$$

Assuming that the elements of the filter are pure reactances then there are four possibilities for the signs of Z_{oc} and Z_{sc}.

1. $Z_{oc} = +j X_a \qquad Z_{sc} = -j X_b$
2. $Z_{oc} = -j X_a \qquad Z_{sc} = +j X_b$
3. $Z_{oc} = +j X_a \qquad Z_{sc} = +j X_b$
4. $Z_{oc} = -j X_a \qquad Z_{sc} = -j X_b$

Substituting in order the four conditions in equation 2.1 it can be seen that conditions 1 and 2 give pure resistance for Z_o while conditions 3 and 4 give a reactive Z_o.

If practical theory is considered it will be noted that if the iterative impedance is pure resistance and the filter components are pure reactive, therefore absorb no power, all the power delivered to the circuit must be absorbed in the terminating resistance. If no power is lost in the filter then the output voltage and current must be equal, respectively, to the input voltage and current. As a consequence there will be no attenuation.

A more elegant method of approaching this same conclusion is by the use of mathematics from the long line theory. The propagation constant is given by:

$$\tanh \gamma = \sqrt{\frac{Z_{sc}}{Z_{oc}}} \qquad 2.2$$

Now if conditions 1 and 2 hold, then

$$\tanh \gamma = j\sqrt{\frac{X_b}{X_a}} \qquad 2.3$$

but when conditions 3 and 4 exist

$$\tanh \gamma = \sqrt{\frac{X_b}{X_a}} \qquad 2.4$$

When the iterative impedance is a pure resistance the hyperbolic tangent of the propagation constant is a pure imaginary, while if Z_o is reactive the propagation constant is a real number. From hyperbolic trigonometry we have:

$$\tanh \gamma = \tanh(\alpha + j\beta) = \frac{\sinh\alpha \cosh\alpha + j\sin\beta \cos\beta}{\sinh^2\alpha + \cos^2\beta} \qquad 2.5$$

α represents the attenuation and β is the phase shift. If $\tanh\gamma$ is imaginary then from equation 2.5, $\sinh\alpha \cosh\alpha$ must be equal to zero. This is possible only if $\alpha = 0$ therefore there will be no attenuation when Z_o is a pure resistance. Conversely there is a finite attenuation when $\tanh\gamma$ is a real number since, then, β must be zero and Z_o is a pure reactance.

If a sketch is made of the open and short circuited reactances the attenuation and transmission bands may be determined. When the reactances are of opposite signs the filter will transmit the frequencies but when the reactances have the same sign attenuation will occur.

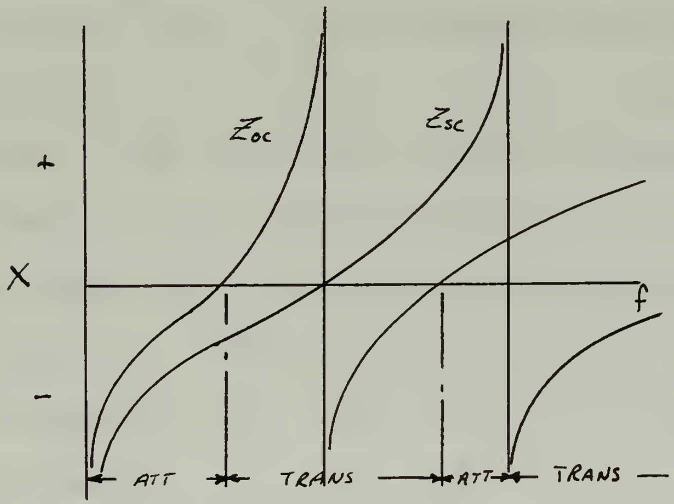


Fig. 2.1 Open and Short-Circuited Reactance Curves

It is laborious to compute the components of a filter using open and short circuit reactances therefore it is more convenient to use the individual arm reactances. Considering the T network in Fig. 2.2 the iterative impedance is given by equation 2.6.

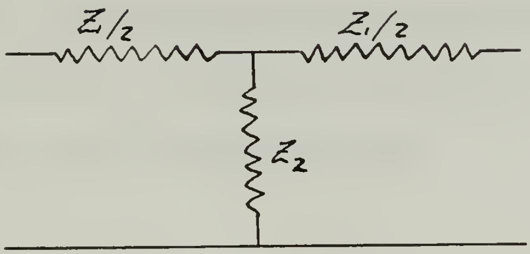


Fig. 2.2 T Network

$$Z_o = \sqrt{Z_1 Z_2 + \frac{Z_1^2}{4}} \qquad 2.6$$

this may be separated as follows:

$$Z_o = \sqrt{Z_1 \left(Z_2 + \frac{Z_1}{4} \right)} \qquad 2.7$$

If the reactance of Z_1 is opposite in sign to the reactance of ($Z_2 + Z_1/4$) the iterative impedance is pure resistance while it is reactive if the two quantities have the same sign. From this the reactance curves of these two quantities will reveal the attenuation and transmission bands in the same way as the open and short circuit impedance curves of Fig. 2.1. The cut-off frequencies occur when the critical frequencies of Z_1 and ($Z_2 + Z_1/4$) do not occur at the same frequency.

For this to be possible:

$$Z_1 = 0 \quad \text{when } Z_2 \neq 0 \text{ or } \infty$$

$$\text{or} \quad Z_2 + Z_1/4 = 0 \quad \text{when } Z_1 \neq 0 \text{ or } \infty$$

$$\text{or} \quad Z_2 = \infty \quad \text{when } Z_1 \neq 0 \text{ or } \infty$$

2.8

Equations 2.8 can be restated in a limit of the ratio for transmission. For Z_o to be real:

$$-1 < Z_1/4Z_2 < 0 \qquad 2.9$$

A sketch of the use of the reactance curves for determination of the transmission and attenuation bands is shown in Fig. 2.3. A typical T network filter is shown alongside the reactance curve.

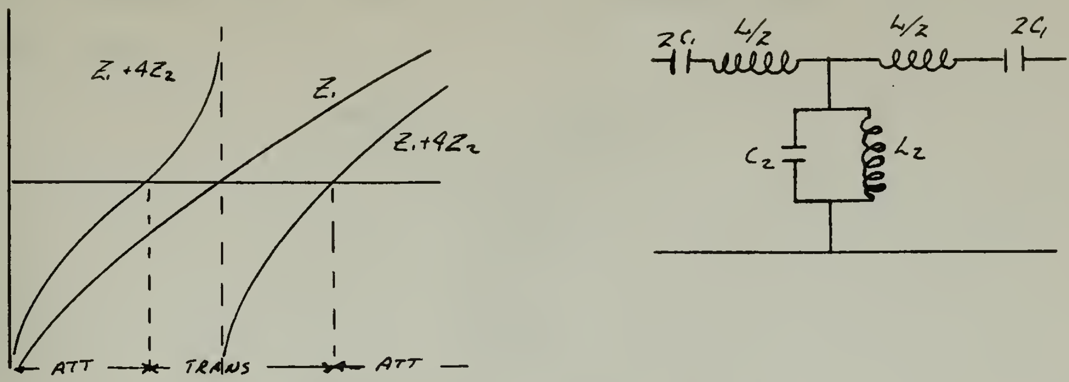


Fig. 2.3 Reactance Curve of T Filter

It is desirable to be able to have a point of infinite attenuation near the cut-off frequency so that a sharp cut-off is experienced. To make this possible the so-called *m*-derived filters were developed by Zobel so that filter sections with different frequencies of infinite attenuation may be joined with matched impedances. If the iterative impedances are matched at all frequencies then all sections would have the same transmission region.

Let equation 2.6 be the basis for the derivation and call it the prototype. In the derived filter let the new branches be called Z_1' and Z_2' with an iterative impedance Z_0'. The impedance Z_1 and Z_1' are now related by the equations:

$$Z_1' = m Z_1 \qquad 2.10$$

Since $Z_0 = Z_0'$ the problem is to solve for Z_2'.

Equating 2.6 to Z_0'

$$Z_1 Z_2 + Z_1^2/4 = Z_1' Z_2' + \frac{Z_1'^2}{4} = m Z Z_2 + \frac{m^2 Z_1^2}{4}$$

Solving for Z_2'.

$$Z_2' = \frac{Z_2}{m} + \frac{(1+m^2)}{4m} Z_1 \qquad 2.11$$

The derived filter now has the configuration shown in Fig. 2.4.

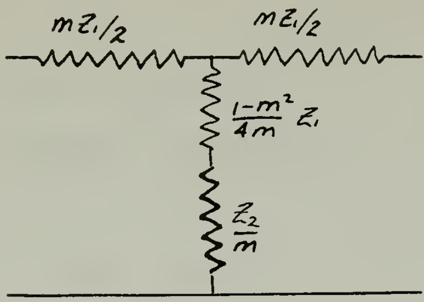


Fig. 2.4 m-derived T Filter

A representative band pass filter derived from that shown in Fig. 2.3b is sketched in Fig. 2.5.

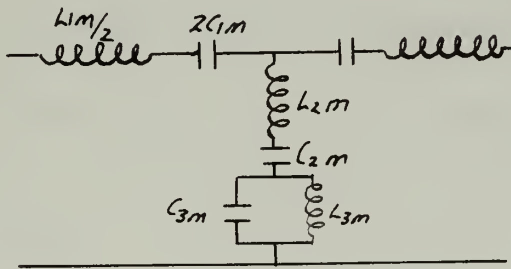


Fig. 2.5 Typical m-derived Filter

The derived values are:

$L_{1m} =$	mL_1	$C_{2m} =$	$4mC_1/(1-m^2)$	
$C_{1m} =$	C_1/m	$L_{3m} =$	L_2/m	
$L_{2m} =$	$(1-m^2)L_1/4m$	$C_{3m} =$	mC_2	2.12

where $m = \sqrt{1-\left(\frac{\omega_\infty(\omega_2-\omega_1)}{\omega_\infty^2-\omega_1\omega_2}\right)^2}$ 2.13

ω_∞ is the frequency of infinite attenuation, ω_1 and ω_2 are lower and upper cut-off frequencies respectively.

Since the crystal filters, to be described in subsequent chapters, are derived from the lattice type filter it is important to consider the properties of this network.

Considering the lattice network of Fig. 2.6a and again returning to equation 2.1, substituting therein, equations 2.14, we have;

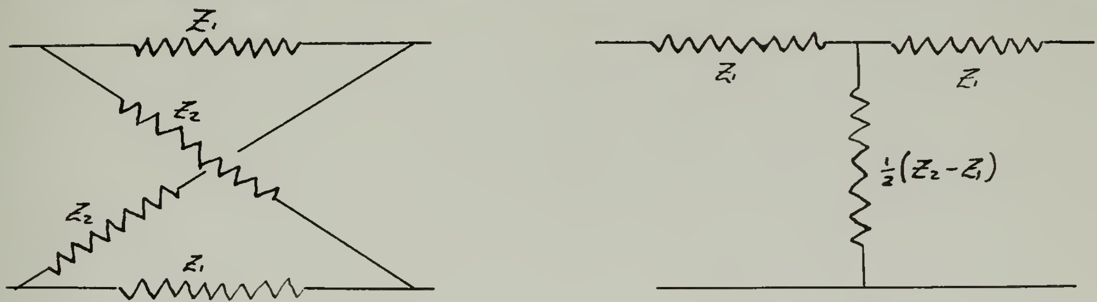
$$Z_{oc} = \tfrac{1}{2}(Z_1 + Z_2)$$

$$Z_{sc} = 2Z_1 Z_2/(Z_1 + Z_2) \qquad 2.14$$

then

$$Z_o = \sqrt{Z_{oc} Z_{sc}} = \sqrt{Z_1 Z_2} \qquad 2.15$$

and it can be seen that the lattice section is equivalent to the T section of Fig. 2.6b. From equation 2.15 it is apparent that if Z_1 and Z_2 are reactances of opposite sign the iterative impedance will be resistive and the filter will transmit, while if the reactances are of the same sign



(a) (b)

Fig. 2.6 Lattice Network and Equivalent T

attenuation will occur since Zo will be reactive. This is similar to the Z_{oc} - Z_{sc} analysis of the T type filter, therefore, the transmission and attenuation bands may be determined from the plot of the reactance curves. The attenuation α and phase shift β may be determined from;

$$\tanh \gamma = \sqrt{\frac{Z_{sc}}{Z_{oc}}} = \frac{2\sqrt{Z_1 Z_2}}{Z_1 + Z_2} = \frac{2Z_o}{Z_1 + Z_2} \qquad 2.16$$

The translation from lattice to T section is not always physically realizable. An example of this is the simple

case where Z_1 is a capacitance, $1/j\omega C$ and Z_2 is an inductance $j\omega L$. It is possible, however, to translate any ladder type network to an equivalent lattice with physically realizable elements.

Lattice sections are not employed as frequently as ladder sections because they require more impedances per section and are less economical for this reason. It does have certain properties which are advantageous for the construction of crystal filters and will be discussed in those chapters dealing with crystal filters.

CHAPTER III

CRYSTALS FOR FILTERS

In the frequency range from 50 to 500 kilocycles the 18.5° X-cut crystal is normally employed in crystal filters. At frequencies above 500 kilocycles it is necessary to go to high frequency shear crystals such as AT and BT cuts. The longitudinal cuts are used and are preferable in these frequency ranges because they have, in general, a single frequency free from secondary resonances due to the fact that the dimension in the direction of vibration is large compared to the other dimensions.

To appreciate the crystal's function in the filter circuit it is necessary to review the nature of the crystal and its equivalent circuit. The equivalent circuit is shown in Fig. 3.1 of a crystal free to vibrate on both ends. From fundamental relation of vibrations of thin

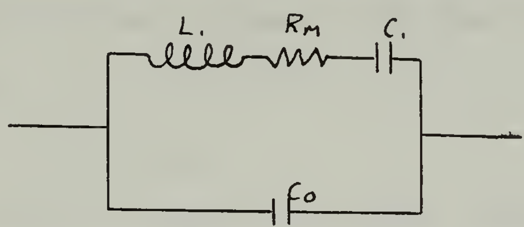


Fig. 3.1 Crystal Equivalent Circuit

plates it can be shown that the element values can be computed from the dimensions and the fundamental constants of the crystal. These equations are;

$$C_o = 0.402 \frac{l_w l_y}{l_t} \ \mu\mu f$$

$$C = 2.924 \times 10^{-3} l_w l_y / l_t \ \text{uuf}$$

$$L = 139 \ l_t l_y / l_w \ \text{henries} \qquad 3.1$$

where the dimensions are in centimeters.

The resistance element represents the mounting resistance, the internal dissipation of the crystal and the radiation loss from the ends of the crystal. In a well mounted crystal the radiation loss is the largest of the losses and it may be calculated from the relation;

$$R_M = (\rho v)_a \, l w / t \qquad 3.2$$

where $(\rho v)_a$ equals 43 ohms per square centimeter in air. Normally this resistance is of little interest since the Q of 18.5° X-cut crystals is of the order of 24,500 in air and considerably higher when carefully mounted in an evacuated container. The Q can be calculated from the vibration of thin plate theory and is;

$$Q = \frac{\pi}{4}\frac{(\rho v)_q}{(\rho v)_a}, \quad (\rho v)_q \text{ is the } (\rho v) \text{ in the crystal}$$

$$= \frac{\pi}{4}\left(\frac{2.65 \times 5.08 \times 10^5}{43}\right) = 24,500$$

From the values of the capacities in the crystal equivalent circuit it is apparent that there is a fixed ratio between the two values. For the filter type crystals, this becomes;

$$r = C_0/C_1 = 138 \qquad 3.3$$

This ratio becomes important in the limitation in the use of crystals in filter circuits since the ratio of the anti-resonance frequency to the resonance frequency is related by the relation;

$$f_A^2 / f_R^2 = 1 + 1/r \qquad 3.4$$

where f_A is the anti-resonant frequency and f_R is the resonant frequency. When crystals are used in filters, two quantities are usually specified, namely, the resonant

frequency of the crystal and the series capacitance. The shunt capacitance is usually incorporated with an electrical capacitance which is derived from the filter equations, to be developed in a later chapter. The resonant frequency is determined principally by the mechanical axis lengths. The capacitance of the series condenser is determined by the ratio of the area to the thickness. Another condition is that the optical axis length should be kept as small as possible to prevent spurious resonances from occuring in the vicinity of the principle resonance.

In crystal filters, split plated crystals are in general usage. Originally they were employed in balanced lattice filters to eliminate two crystals that were identical. By dividing the plating an undesirable effect occurs, namely, that the low frequency flexure mode will be driven. Referring to Fig. 3.2 this is due to the elongation at the bottom. Normally this mode is too low in frequency to interfere with the longitudinal mode and can be neglected. It can, however, be troublesome, as will be pointed out in Chapter IV, therefore crystals should be carefully selected before being placed in a filter network.

To illustrate that the diagonal plates will not excite longitudinal vibration, the approximate equivalent circuit of a split plated crystal is shown in Fig. 3.2b.

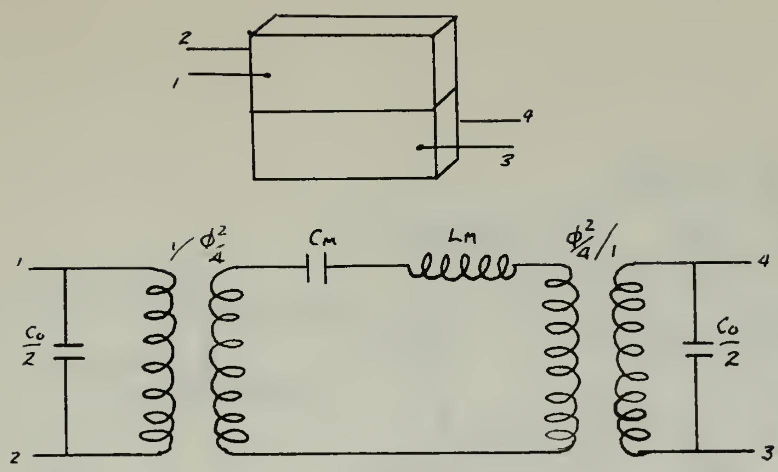


Fig. 3.2 Split Plated Crystal and Equivalent Circuit

The inductance and capacitance are as described before and $\phi^2/4$ is the impedance transformation ratio. Note that the static capacitance C_o is halved due to the reduction of the plate area by two.

The four methods of connecting a crystal are shown in Fig. 3.3 with their equivalent electrical circuits. It can be readily seen that they lend themselves to a balanced lattice configuration. In the filters constructed and tested the configurations A and B were used since these arrangements placed the largest plating capacitance in the same arm as the crystal and minimized the capacitance in the other arms. Proof of these circuits are rather lengthy and are well presented by reference 9 so that no attempt will be made to include it here.

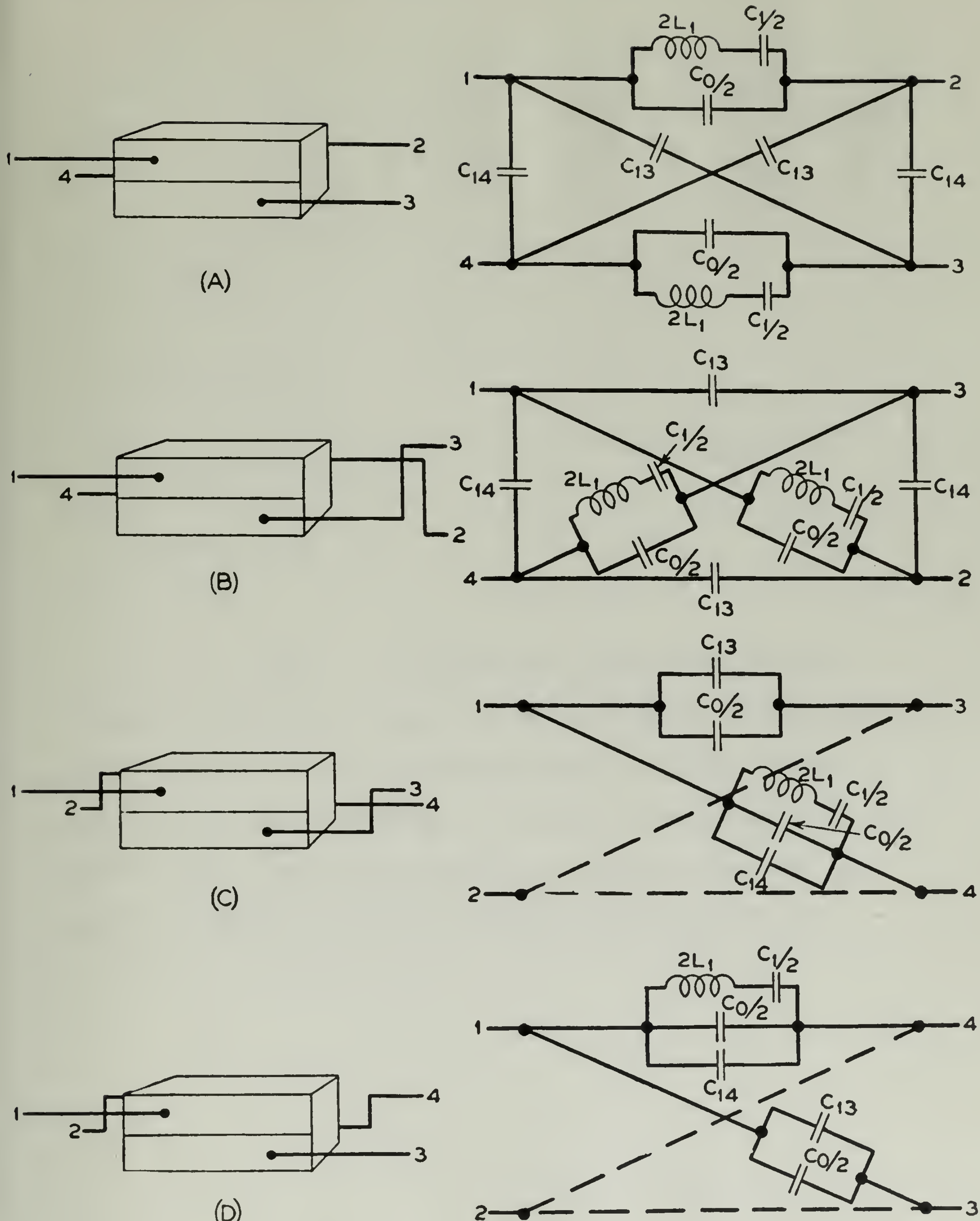


Fig. 3.3 Methods for Connecting Crystals

If an unbalanced filter is desired the split plated type crystals may be connected as shown in Fig. 3.4.

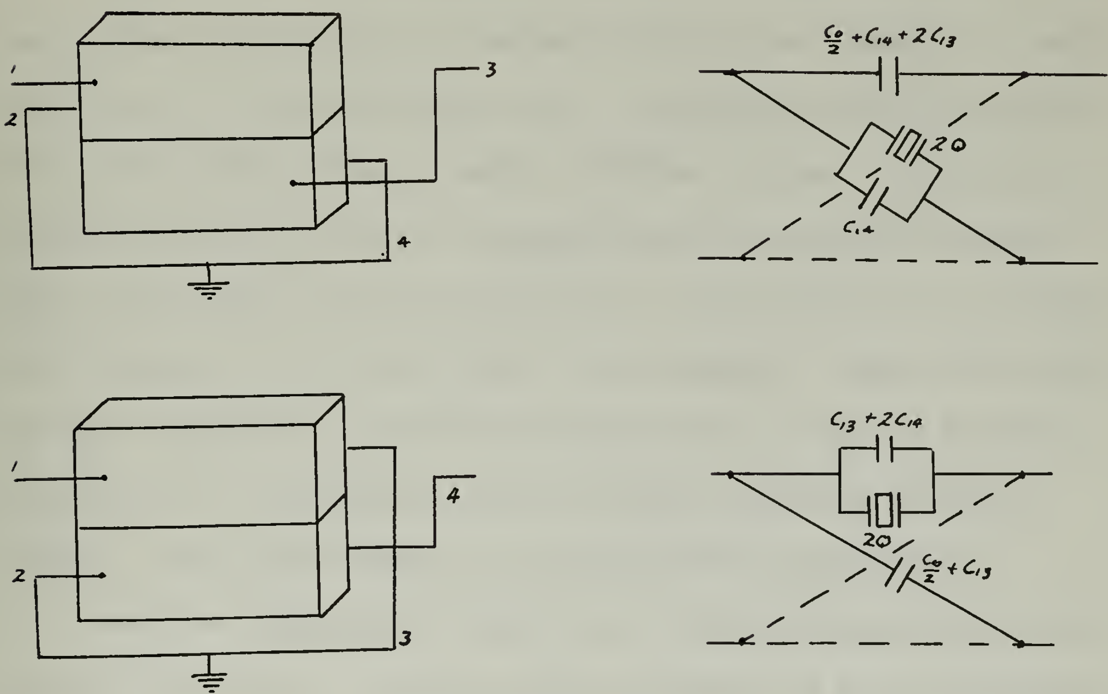


Fig. 3.4 Crystal for Unbalanced Filter

An unbalanced filter tends to have its total band width limited due to the unbalancing effect of $2C_{13}$ in the series arm which is not balanced by a similar capacitance in the lattice arm. Shielding strips are used to connect the common grounded plates to minimize this capacitance.

CHAPTER IV

LOW IMPEDANCE CRYSTAL FILTERS

The basic development, as has been pointed out, has been done primarily by the Bell Telephone Company to make possible a filter suitable for selecting certain desirable frequency bands from a cable carrier system. Since their primary interest was in terminating the existing cables with the crystal filters they were interested in a filter that matched the cable over the frequency range for which it was designed. For this reason most of the literature available is concerned with the low impedance crystal filter to be described in the following paragraphs.

The low impedance filter has the configuration shown in Fig. 4.1 with a series coil in each arm and a capacitor

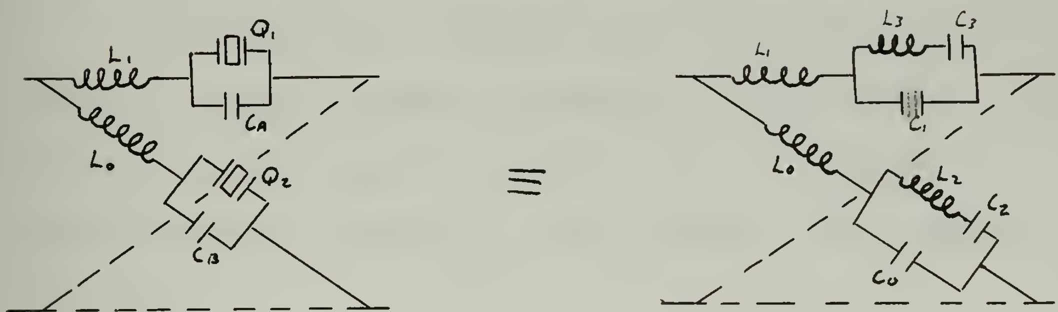


Fig. 4.1 Series Coil Crystal Filter

in parallel with the crystal. The sum of the reactances of the coil and crystal series circuit is given by;

$$X = j\left[\omega L_1 - \frac{1}{\omega(C_1+C_3)}\left(1-\frac{\omega^2/\omega_R^2}{\omega^2/\omega_A^2}\right)\right] \qquad 4.1$$

where

$$\omega_R^2 = \frac{1}{L_3 C_3} \qquad \text{(resonant angular frequency)}$$

and

$$\omega_A^2 = \frac{(C_1+C_3)}{L_3 C_3 C_1} = \omega_R^2\left[1+\frac{C_3}{C_1}\right] \qquad \text{(anti-resonant angular frequency)}$$

If the reactances are added and the numerator expressed as a factor

$$X = \frac{-j}{\omega(C_1+C_3)}\left\{\frac{1-\omega^2\left[L_1C_1+C_3(L_1+L_3)\right]+\omega^4L_1L_3C_1C_3}{1-\omega^2/\omega_A^2}\right\} \qquad 4.2$$

let

$$\omega_1^2 = \frac{1}{2}\left\{\frac{\left[L_1(C_1+C_3)+L_3C_3-\sqrt{\left[L_1(C_1+C_3)-L_3C_3\right]^2-4L_1L_3C_1C_3}\right.}{L_1L_3C_1C_3}\right\} = \frac{1}{2}\left[\omega_A^2-\omega_\alpha^2-\sqrt{(\omega_A^2+\omega_\alpha^2)^2-4\omega_\alpha^2\omega_R^2}\right]$$

where $\omega_\alpha^2 = \frac{1}{L_1C_1}$ (resonance of series coil and C)

$$\omega_2^2 = \omega_A^2$$

and

$$\omega_3^2 = \frac{1}{2}\left\{\frac{\left[L_1(C_1+C_3)+L_3C_3+\sqrt{\left[L_1(C_1+C_3)+L_3C_3\right]^2-4L_1L_3C_1C_3}\right.}{L_1L_3C_1C_3}\right\}$$

$$= \frac{1}{2}\left[\omega_A^2+\omega_\alpha^2+\sqrt{(\omega_A^2+\omega_\alpha^2)^2-4\omega_\alpha^2\omega_R^2}\right]$$

then

$$X = \frac{-j}{\omega(C_1+C_3)}\left[\frac{(1-\omega^2/\omega_1^2)(1-\omega^2/\omega_3^2)}{1-\omega^2/\omega_2^2}\right] \qquad 4.3$$

If the product of $\omega_1\omega_3$ is taken it is apparent that

$$\omega_1\omega_3 = \omega_\alpha\omega_R \qquad 4.4$$

It is desirable to have the resonances symmetrical about the anti-resonant frequency of the crystal, therefore it is necessary to have L_1 and C_1 resonate at f_A the anti-resonant frequency of the crystal. For this to be possible

$$f_1^2 = f_A^2\left[1-\sqrt{1-\omega_R^2/\omega_\alpha^2}\right] = f_A^2\left[1-\sqrt{\frac{C_3}{C_1}}\right]$$

similarly

$$f_3^2 = f_A^2\left[1+\sqrt{\frac{C_3}{C_1}}\right] \qquad 4.5$$

For 18.5° X-cut crystals $C_1/C_3 = r = 138$, therefore the two resonances will occur at

$$f_1 = .9542f_A \text{ and } f_3 = 1.0415f_A \qquad 4.6$$

Now it can be seen that by placing an inductance in series with the crystal two resonances will occur, one due to the

crystal alone and one due to the coil and crystal capacitance. Observing the ratio

$$f_3/f_1 = 1.0915 \qquad 4.7$$

it is seen that the resonances are separated 9.15 per cent of the mean frequency.

The low impedance band pass filter utilizes this property in each arm of the filter. A curve of the reactances in the series and lattice arms is shown in Fig. 4.2. The solid curve represents the series arm reactance and the dotted curve represents the lattice arm reactance. To satisfy the condition for band pass in a lattice filter

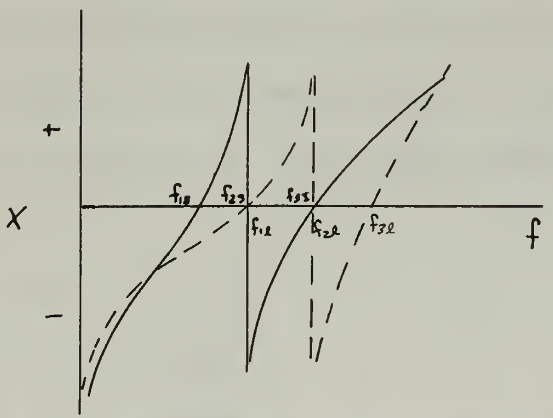


Fig. 4.2 Reactance Curves of Low Impedance Filter

the reactances of the two arms must have opposite signs throughout the pass region. To meet this and the symmetry condition the lowest resonance of the lattice arm must coincide with the anti-resonance of the series arm and the highest resonance of the series arm must coincide with the anti-resonance of the lattice arm.

The ratio between the highest resonance in the lattice arm and lowest resonance in the series arm is

$$f_{3l}/f_{1s} = 1.0915 \times 1.0415 = 1.137$$

giving a maximum pass band that, theoretically is 13.7% of the mean frequency.

The elements of the filter may be determined from the lower and upper cut-off frequencies f_{1s} and f_{3l} respectively, (to be called f_1 and f_3 in subsequent equations), the iterative impedance, Z_0, and the quantity m, whose relation to the cut-off and infinite attenuation frequencies is given in equation 4.8.

$$m = \sqrt{\frac{1 - \omega_\infty^2/\omega_3^2}{1 - \omega_\infty^2/\omega_1^2}} \qquad 4.8$$

To obtain the critical frequencies for a three section lattice filter it is necessary to take the hyperbolic tangent of the sum of the three sections.

Then:

$$m_1 = \frac{1 - \omega_{\infty 1}^2/\omega_3^2}{1 - \omega_{\infty 1}^2/\omega_1^2}$$

$$m_2 = \frac{1 - \omega_{\infty 2}^2/\omega_3^2}{1 - \omega_{\infty 2}^2/\omega_1^2} \qquad 4.9$$

$$m_3 = \frac{1 - \omega_{\infty 3}^2/\omega_3^2}{1 - \omega_{\infty 3}^2/\omega_1^2}$$

and $A = m_1 + m_2 + m_3$

$$B = m_1 m_2 + m_1 m_3 + m_2 m_3 \qquad 4.10$$

$$C = m_1 m_2 m_3$$

The critical frequencies become (refer to Fig. 4.2)

$$f_{1l}^2 = f_{2s}^2 = \frac{f_1^2 f_3^2 (1 + B)}{f_1^2 + f_3^2 B} \ ; \qquad f_{2l}^2 = f_{3s}^2 = \frac{f_1^2 f_3^2 (A + C)}{A f_1^2 + C f_3^2} \qquad 4.11$$

the propagation constant is

$$\tanh \gamma_1 + \gamma_2 + \gamma_3 = \frac{(A+C)}{(1+B)} \sqrt{\frac{(1 - \omega^2/\omega_1^2)(1 - \omega^2/\omega_{2l}^2)^2}{(1 - \omega^2/\omega_{1l}^2)(1 - \omega^2/\omega_3^2)}} \qquad 4.12$$

and the image impedance is

$$Z_s = \sqrt{\frac{-(1 - \omega^2/\omega_1^2)(1 - \omega^2/\omega_3^2)}{\omega^2 (C_1 + C_3)(C_0 - C_2)}} \qquad 4.13$$

at the mean frequency the image impedance becomes

$$Z_{IO} = \frac{(f_3 - f_1)}{2\pi f_3 f_1 \sqrt{(C_0+C_2)(C_1+C_3)}} \qquad 4.14$$

and at zero frequency the impedance ratio of the two arms becomes

$$\frac{Z_1}{Z_2} = \sqrt{\frac{C_0+C_2}{C_1+C_3}} = \tanh(\gamma_1 + \gamma_2 + \gamma_3) = \frac{A+C}{1+B} \qquad 4.15$$

By manipulation of equations 4.2, 4.3, 4.11, 4.14 and 4.15 the design equations in Table I are obtained.

In the practical design of filters it is sometimes desirable to have the last peak come at infinity, in which case the ratio $f_1/f_3 = m_3$ and L_0 and L_1 are then equal.

By a network theorem that states that if equal impedances occur in both the series and lattice arms of the filter these impedances may be brought out and placed in series with the terminal leads. Fig. 4.3 gives the equivalence

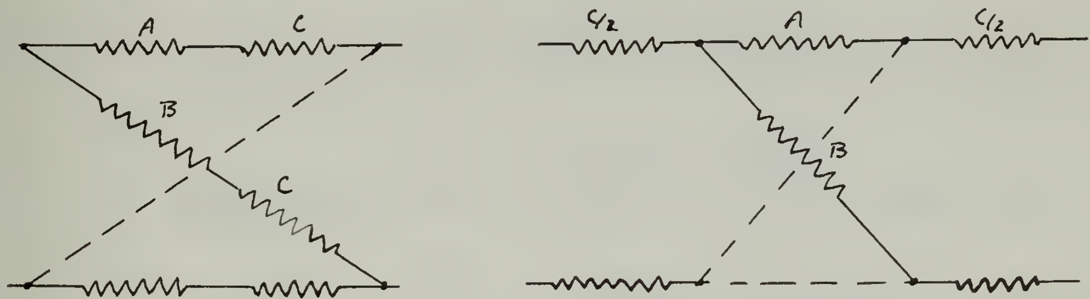


Fig. 4.3 Lattice Network Equivalences

By the use of this theorem the inductors may be made external to the filter and the filter takes the configuration shown in Fig. 4.4.

TABLE I

DESIGN FORMULAE FOR WIDE BAND CRYSTAL FILTER

Element	Formula
L_0........	$\dfrac{Z_0 f_B(f_A{}^2 A + f_B{}^2 C)}{2\pi f_A(f_B - f_A)(f_A{}^2 + f_B{}^2 B)}$
L_1........	$\dfrac{Z_0 f_A(f_A{}^2 + f_B{}^2 B)}{2\pi f_B(f_B - f_A)(f_A{}^2 A + f_B{}^2 C)}$
L_2........	$\dfrac{Z_0[f_B{}^4[B(A + C) - C] + 2f_A{}^2 f_B{}^2 C + f_A{}^4 A]^2}{2\pi f_A f_B(f_B - f_A)^3(f_B + f_A)^2[f_A{}^2 + Bf_B{}^2][AB - C]}$
L_3........	$\dfrac{Z_0[f_B{}^4 BC + 2f_A{}^2 f_B{}^2 + f_A{}^4]^2(A(1 + B) - C)}{2\pi f_A f_B(f_B - f_A)^3(f_B + f_A)^2(Af_A{}^2 + Cf_B{}^2)C(AB - C)}$
C_0........	$\dfrac{(f_B - f_A)(f_A{}^2 + Bf_B{}^2)^2}{2\pi Z_0 f_A f_B(f_B{}^4(B(A + C) - C) + 2f_A{}^2 f_B{}^2 C + f_A{}^4 A)}$
C_1........	$\dfrac{(f_B - f_A)(Af_A{}^2 + Cf_B{}^2)^2}{2\pi Z_0 f_A f_B[f_B{}^4 BC + 2f_A{}^2 f_B{}^2 C + f_A{}^4(A(1 + B) - C)]}$
C_2........	$\dfrac{(AB - C)(f_B - f_A)^3(f_B + f_A)^2}{2\pi Z_0 f_A f_B[f_B{}^4(B(A + C) - C) + 2f_A{}^2 f_B{}^2 C + f_A{}^4 A](A + C)}$
C_3........	$\dfrac{C(AB - C)(f_B - f_A)^3(f_B + f_A)^2}{2\pi Z_0 f_A f_B[f_B{}^4 BC + 2f_A{}^2 f_B{}^2 C + f_A{}^4(A(1 + B) - C)](1 + B)}$

Note: f_A and f_B are synonomous with f_1 and f_2 respectively in text.

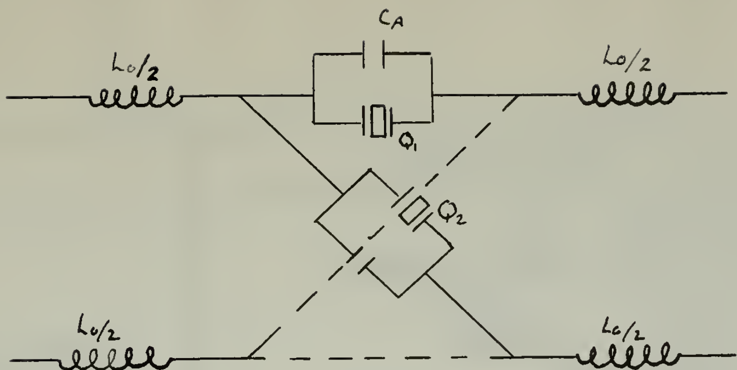


Fig. 4.4 Wide Band Crystal Filters with External Coils

To conserve on elements the split plated crystals described in Chapter II may be used. Also when space is at a premium the two balanced coils can be obtained from a single coil with balanced windings. The final filter consists of 2 coils, 2 crystals and four small capacitors. The physical construction of the filter is shown in Fig. 4.5 along with its electrical equivalent.

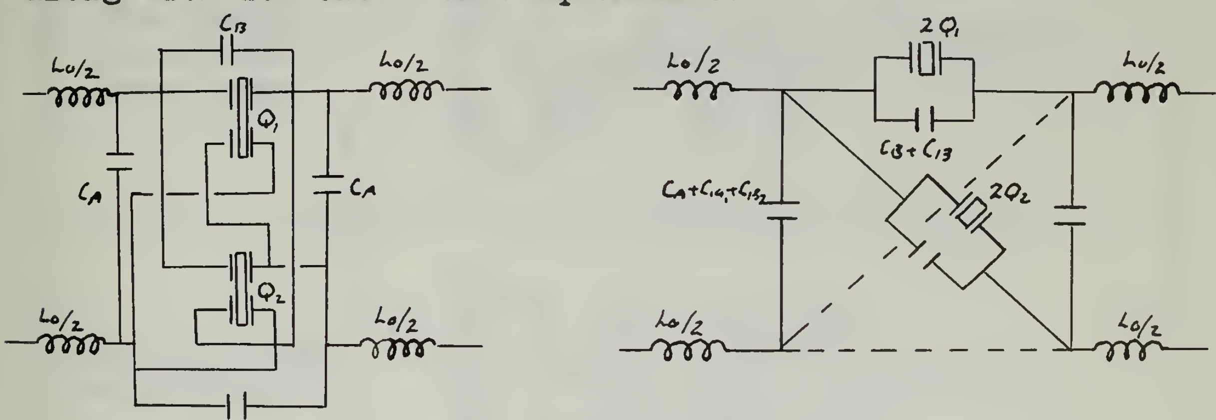


Fig. 4.5 Filter using Split-Plated Crystals

A sketch of the attenuation characteristcs and of the iterative impedance is shown in Fig. 4.6.

A filter of this type was constructed and the results and values of components are described in Chapter VI.

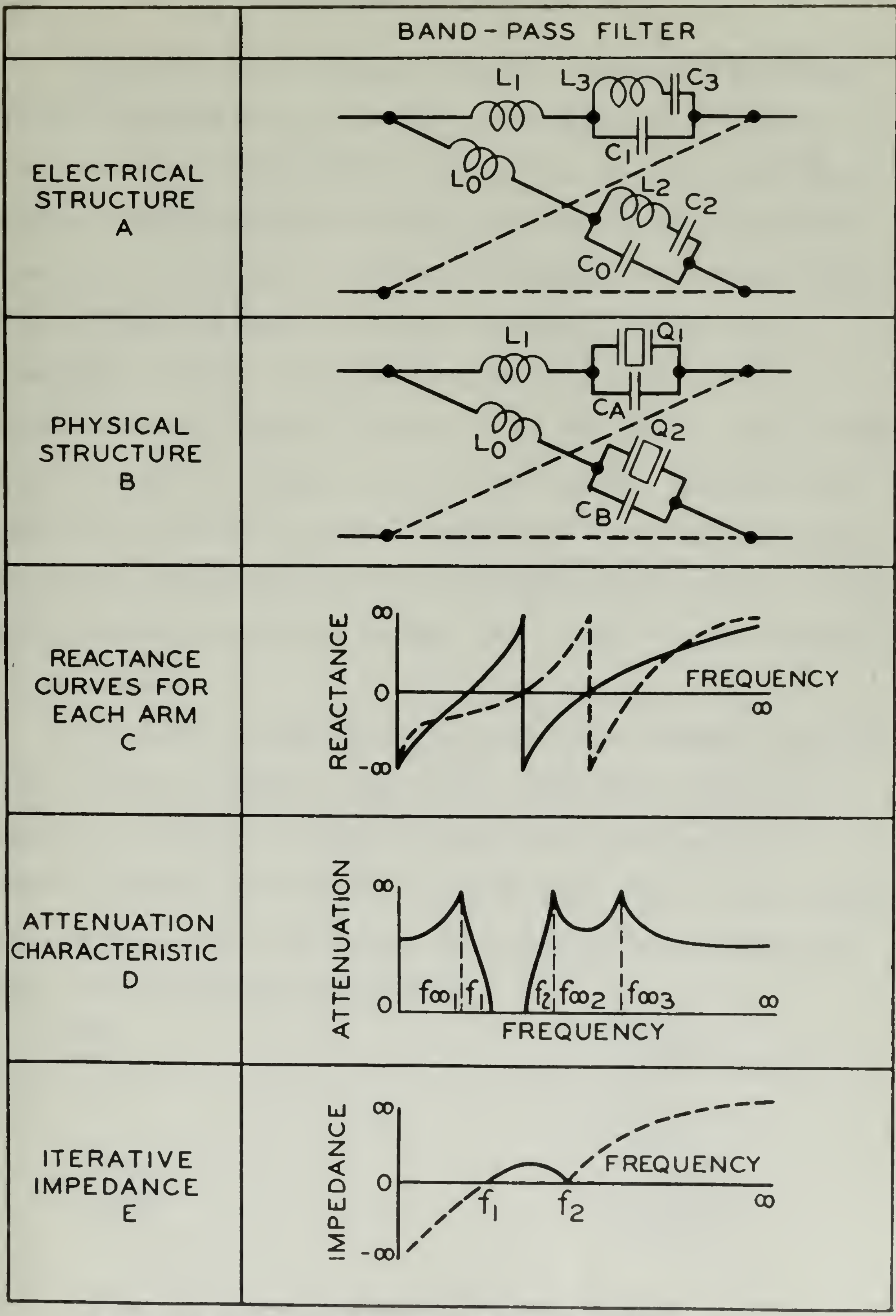


Fig. 4.6 Crystal Filter Employing Series Coils

CHAPTER V

HIGH IMPEDANCE CRYSTAL FILTERS

Since the low impedance filter's iterative impedance becomes approximately 25 ohms at 500 kilocycles its use is precluded in vacuum tube circuits. In the past year there has been considerable work done by many manufacturers of electronic equipment to develop a subminiature high impedance filter for use as a very selective intermediate frequency filter. The few texts on the subject tend to gloss over this type of filter as it had little application until recently. These filters are a bridge configuration that are balanced at their frequencies of infinite attenuation and are unbalanced for all other frequencies. In the pass band the unbalance is such that the iterative impedance is resistive.

The basic configuration for the high impedance lattice type filter is shown in Fig. 5.1. Note that the primary physical difference between this filter and the low impedance filter is that the inductors are now in shunt with the crystal. The electrical characteristics, as will be seen, are considerably different.

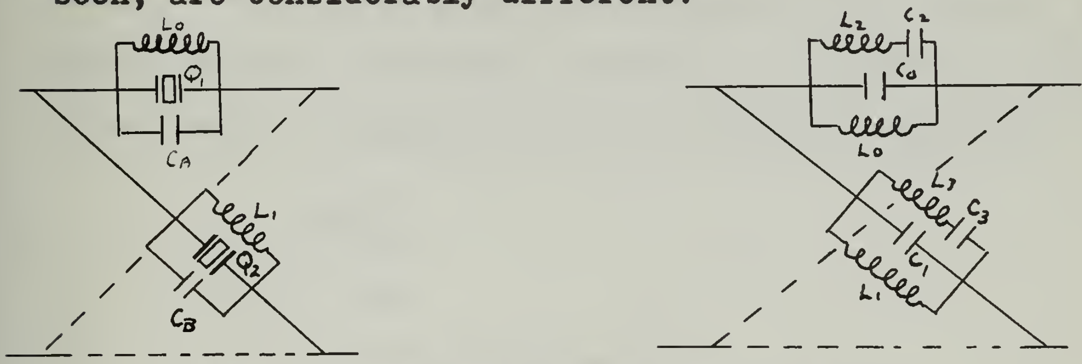


Fig. 5.1 High Impedance Crystal Filter

The sum of the reactances of, say, the series arm of the filter is

$$Z = \frac{1}{j\omega C_0 + \frac{1}{j\omega L_0} + \frac{1}{j\omega L_2 + \frac{1}{j\omega C_2}}} \qquad 5.1$$

this reduces to

$$Z = \frac{-j(\omega L_0(1-\omega^2/\omega_R^2)}{\omega^2 L_0 C_2 - (1-\omega^2/\omega_A^2)(1-\omega^2/\omega_R^2)} \qquad 5.2$$

where

$$\omega_A^2 = \frac{1}{L_0 C_0} \qquad \omega_R^2 = \frac{1}{L_2 C_2}$$

Referring to curves of the reactances of each arm shown in Fig. 5.2, it is noted that the first pole of the lattice arm coincides with the first zero (other than

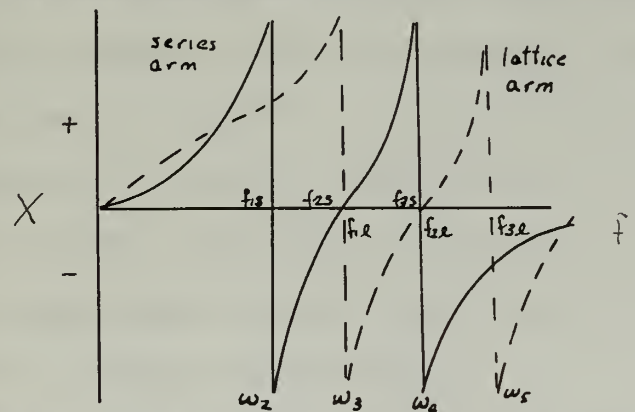


Fig. 5.2 Reactance Curves for Filter of Fig. 5.1

zero frequency) of the series arm and the second pole of the series arm coincides with the first zero of the lattice arm. These frequencies should be evenly spaced between the region from f_{1s} to $f_{3\ell}$ for proper operation of the filter. By the use of Foster's reactance theorem the components of the filter may be computed from the following equations;

$$L_0 = \frac{H\omega_3^2}{\omega_2^2\,\omega_4^2}$$

$$C_0 = 1/H$$

$$L_3 = \omega_3^2 H/(\omega_3^2-\omega_2^2)(\omega_3^2-\omega_4^2)$$

$$C_3 = \frac{1}{\omega_3^2 L_3}$$

where

$$H = \frac{Z_0(\omega_m^2-\omega_2^2)(\omega_m^2-\omega_4^2)}{j\,\omega_m(\omega_m^2-\omega_3^2)}$$

Z_0 is the iterative impedance at ω_m, the mean frequency of the filter.

$$\omega_m = \sqrt{\omega_2\,\omega_5}$$

The method most commonly used to obtain the component values is by employing equation 5.2 and the developed equations, 4.11 to 4.14, of Chapter IV. The manipulation of these equations follows the same scheme as that of Chapter IV so the results are listed in Table II. The use of these equations take into account the location of the frequencies of infinite attenuation whereas the direct application of Foster's equations do not, unless the value of H is chosen separately for each arm so the reactance of the two arms have the same sign and are equal at the frequency of infinite attenuation.

If the Table II values are used and m is chosen so that the third frequency of infinite attenuation lies at infinity, that is, $m_3 = f_1/f_3$, the shunting inductors may be brought out external to the filter as in the case of the low impedance filter since L_0 and L_1 are equal. Schematically this is shown in Fig. 5.3. The application of this theorem makes possible the use of balanced filters in intermediate frequency sections of radio receivers.

TABLE II

Design Formulae for Crystal Filter With Shunt Coils

Element	Formula
L_0	$\dfrac{Z_0(f_B - f_A)(1 + B)}{2\pi f_A f_B(A + C)}$
L_1	$\dfrac{Z_0(f_B - f_A)(A + C)}{2\pi f_A f_B(1 + B)}$
L_2	$\dfrac{Z_0(1 + B)(f_A^2 + Bf_B^2)^2}{2\pi f_A f_B(f_B - f_A)(f_B + f_A)^2(AB - C)}$
L_3	$\dfrac{Z_0(A + C)(Af_A^2 + Cf_B^2)^2}{2\pi f_A f_B(f_B - f_A)(f_B + f_A)^2 C(AB - C)}$
C_0	$\dfrac{(Af_A^2 + Cf_B^2)f_B}{2\pi Z_0 f_A(f_B - f_A)(f_A^2 + Bf_B^2)}$
C_1	$\dfrac{f_A(f_A^2 + Bf_B^2)}{2\pi Z_0 f_B(f_B - f_A)(Af_A^2 + Cf_B^2)}$
C_2	$\dfrac{(f_B - f_A)(f_B + f_A)^2(AB - C)}{2\pi Z_0 f_A f_B(f_A^2 + Bf_B^2)(1 + B)^2}$
C_3	$\dfrac{(f_B - f_A)(f_B + f_A)^2 C(AB - C)}{2\pi Z_0 f_A f_B(A + C)^2(Af_A^2 + Cf_B^2)}$

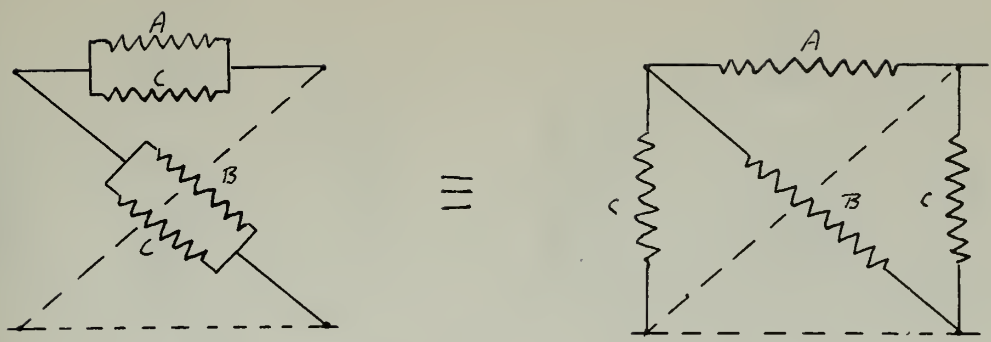


Fig. 5.3 Lattice Network Equivalence

By virtue of the fact that the coils may be removed from shunting the crystals the effect of the losses in the inductors is minimized. If the coils are in the arms of the filter the dissipation varies with the frequency since Q is a function of frequency. When the coils are placed external to the filter the source of dissipation is removed and the result is a constant loss over the pass band of the filter. These filters as all filters, must be properly terminated to prevent reflections, therefore the resistance of the coils may be subtracted from the termination resistance. A practical application of a high impedance filter is discussed in the following chapter.

Since the lattice filter is a bridge network the inherent difficulty of stray capacitances in the high impedance type bridge, of unbalancing, the network, is experienced. Care must be taken in the construction of the filter to either equalize these stray capacitances or eliminate them.

A sketch of the construction of a shunt coil, high impedance type crystal filter is shown in Fig. 5.4.

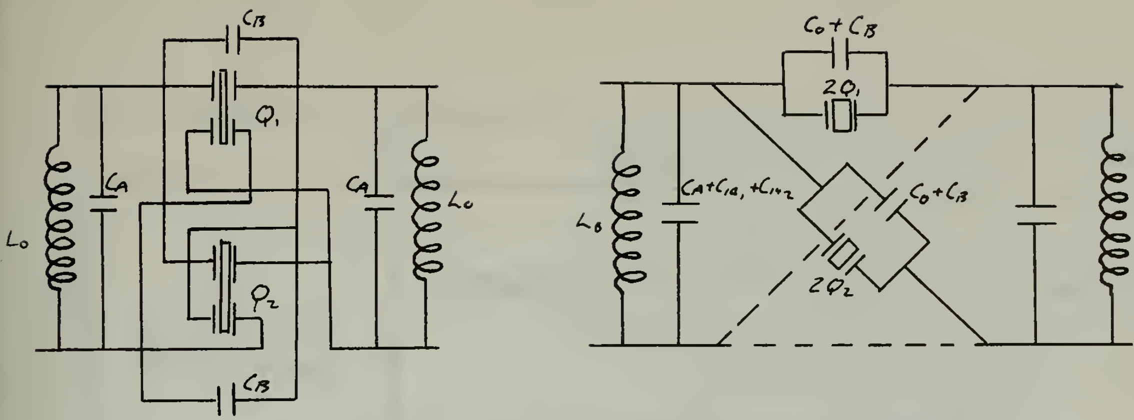


Fig. 5.4 High Impedance Crystal Filter

By bringing out the inductors two coils may be eliminated thus reducing the size and cost of the filter.

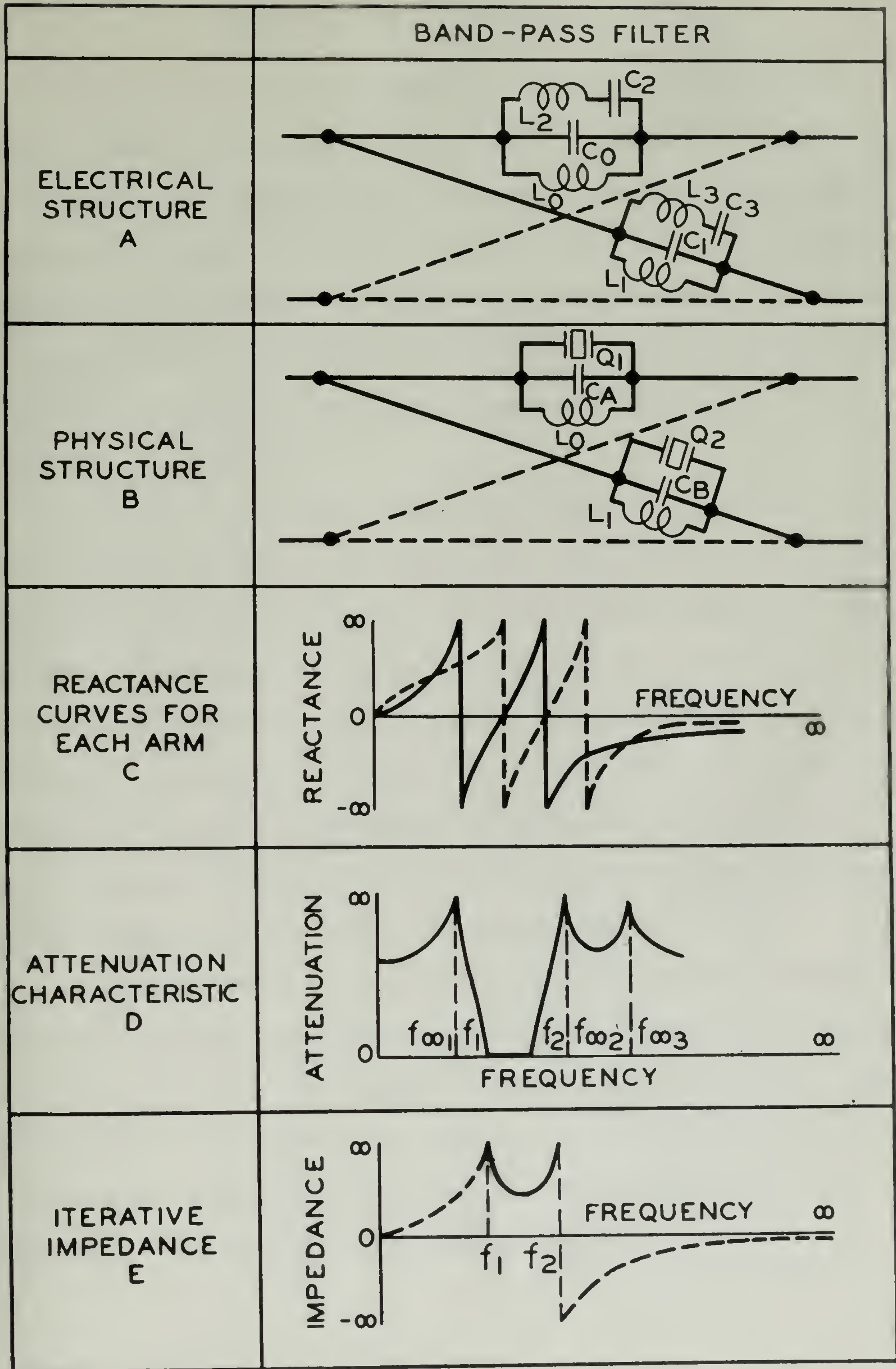


Fig. 5.5 Crystal Filter Employing Shunt Coils

CHAPTER VI

FILTER CONSTRUCTION

Several filters were constructed during the investigation conducted at Bendix Radio of crystal wave filters. Considerable difficulty was experienced in obtaining the desired pass band and sufficient attenuation outside the pass band; therefore it is felt that the experience gained from the study and construction of these filters should be passed on to whomever may find occasion to construct such a filter in the future.

The ultimate use in which the filter was employed was an intermediate frequency band pass filter with a center frequency of 135 kilocycles. The specifications were that the complete receiver was to be not more than 6 db down at ± 2.5 kilocycles from center frequency and at least 60 db down at ± 5 kilocycles. Since the filter was to be used between two vacuum tube stages it was necessary that the main interest should be in the shunt coil or high impedance type filter to provide proper loading for the tubes.

A test set up was devised for testing these filters so that a signal generator with an unbalanced output could be used along with an unbalanced vacuum tube voltmeter for detection. The circuit was designed on the lines of the intermediate frequency amplifier section that was to be employed. The circuit is shown in Fig. 6.1.

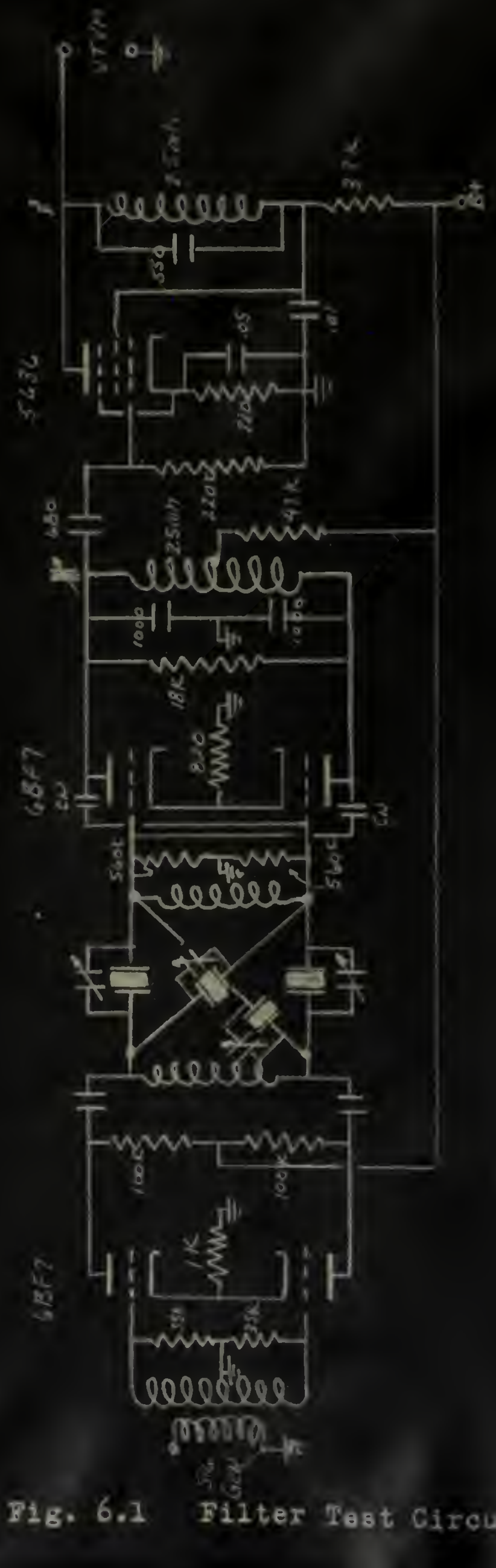


Fig. 6.1 Filter Test Circuit

The first filter was inserted in the test set up with the designed values for the components. Much difficulty was experienced in getting anything that looked like a pass band in the region desired. A restudy of the theory made it apparent that, if the individual arm's reactance curves were adjusted so that they took on the appearance of Fig.5.2 Chapter V, the filter should perform properly. Accurate impedance measuring equipment was not available so it was necessary to measure the reactance with a Q-meter. Curves were taken and the reactance was adjusted in accordance with theory. These curves are shown on page 48, Appendix B. To take the data for these curves it was necessary to disconnect each arm individually and measure reactance.

Reconnecting the filter it was found that the pass band still was not acceptable. Study revealed that the reactance of each arm removed from the filter was not the reactance when it was connected due to the inter-electrode capacitances of the other arms not being in place during the measurements. Another fact revealed, although not fully appreciated at the time the analysis was being done, was that the resonances were not equally spaced in the manner predicted by theory.

Because of the nature of the high impedance filter being easily unbalanced by stray capacitances it was decided to construct a low impedance filter which, it was felt, would be easier to make operate satisfactorily. The design calculations were made, Appendix A, and the filter

was constructed. The reactance curves were adjusted, as before, for the proper coincidence of resonance and anti-resonant frequencies of the two arms. As before, it was found that the pass band didn't exist. A thought occurred at this time to investigate the possibility of using open and short circuited impedances for the determination of the pass band. This was attempted and found to be the answer in this case. Only slight additional adjustment of the variable capacitors was necessary to get the data for the curve shown on page 50, Appendix B.

With the encouragement from the successful operation of the low impedance filter it was again attempted to construct the high impedance filter using the same method of approach for adjusting it to its pass band characteristic. Since the Q-meter was the impedance measuring device it was exceedingly difficult to obtain measurements of the high impedances with accuracy. The method was to choose a frequency in the pass band and adjust the filter to be inductive for the open circuit reactance then check to see if it was capacitive for the short circuited reactance. This was a tedious process and a hit or miss proposition at the best. Again an approximate solution was achieved and the filter was replaced in the test set up. With the high impedance filter it was found that the final adjustment was not as easily achieved.

It was suggested, at this point in the investigation, that the termination of the filter might be causing part

of the difficulty. From the impedance measurements taken, the iterative impedance was calculated, and found to be 1.2 megohms at 135 kilocycles. The filter was terminated in this resistance and after considerable adjustment the data for curve, on page 51, Appendix B was taken.

In the latter stage of the adjustment the effect of each adjustment of the capacitors was recorded. Each pair of capacitors was adjusted together so that the arms impedance would remain equal. After the desired attenuation characteristics were obtained and the component values were measured it was found that the components were not equal in value as was expected. The reason for the difference was assumed to be the difference in the stray capacitance of the bridge structures.

An investigation of the effect of the termination was made and it was found that for each termination the filter had to be readjusted as the attenuation characteristcs changed with termination. The computed termination was found to be the optimum although it could be varied about ±25% in the region of 1.2 megohms without much deterioration in the attenuation curve.

The irregularities in the pass band portion of the curve gave some trouble in explaining the reason for less attenuation at one point than at the other. The solution came just as the project was completed and time was not available for rerunning of the data. The reason was that the second single tuned amplifier was being detuned 10 kilo-

cycles from the mean frequency by the placement of the vacuum tube voltmeter across the tuned circuit.

To verify the theory that permits the inductors to be moved from shunting the crystal to the input and output circuits, these inductors were moved. It was found that the circuit was somewhat more difficult to adjust but it could be done and the theory was proven.

In conclusion the following steps are outlined for guidance in constructing a high impedance broad band crystal lattice filter:

(a) Select mean frequency (f_m) of pass band making certain that desired pass band is less than 13.5% of the mean frequency.

(b) Referring to Fig. 5.2, where f_1 and f_3 are the lower and upper cut-off frequencies respectively, compute f_{2s} (ω_3) and f_{3s} (ω_4) from equations 4.11, Chapter IV.

(c) Select 18.5° X-cut crystals whose resonant frequencies are f_{2s} and f_{3s}. Make sure these crystals have no spurious responses near the pass band.

(d) Compute or measure the crystal's equivalent component values. Actually only one component value need be computed for use in further computations.

(e) From the value or values measured or computed in (d) determine from its appropriate equation the iterative impedance of the filter using Foster's equations or the equations in TABLE II. If TABLE II is used the values of m_1, m_2, and m_3 must be calculated from the predetermined

frequencies of infinite attenuation.

(f) Using the computed Z_o, calculate the remaining components in the filter structure.

(g) Construct filter making certain in initial construction to keep the stray capacitances minimized by carefull placement of parts.

(h) By means of a suitable impedance measuring instrument make the initial open and short circuit impedance adjustments so that these impedance will have opposite signs in the pass band and the same signs outside the pass band. A methodical record of the adjustments should be made since there are four parameters being adjusted.

(i) When the impedance is adjusted as closely as possible run attenuation curves, either manually or with a sweep signal generator and oscilloscope, making final adjustments as necessary.

(j) If the coils are to be taken outside the filter make this change after the values of the circuit parameters have been measured and recorded. Only fairly simple adjustments should then be necessary to retune the filter.

From the experience gained from actual construction of the filters it was found that the most important steps were the selection of the crystal frequencies and the selection of crystals without spurious resonances. The adjustments were tedious but when a methodical approach is used much time can be saved and successful operation of the filter can be obtained.

APPENDIX A

HIGH IMPEDANCE FILTER DESIGN CALCULATIONS

Let $f_1 = 133$ kc $\quad f_{\infty 1} = 130$ kc $\quad f_{\infty 3} = 144$ kc

$f_3 = 137$ kc $\quad f_{\infty 2} = 140$ kc

$$m_1 = \sqrt{\frac{1 - f_{\infty 1}^2/f_3^2}{1 - f_{\infty 1}^2/f_1^2}} = \sqrt{\frac{1 - .9}{1 - .956}} = 1.51$$

$$m_2 = \sqrt{\frac{1 - f_{\infty 2}^2/f_3^2}{1 - f_{\infty 2}^2/f_1^2}} = \sqrt{\frac{1 - 1.03}{1 - 1.11}} = .523$$

$$m_3 = \sqrt{\frac{1 - f_{\infty 3}^2/f_3^2}{1 - f_{\infty 3}^2/f_1^2}} = .343$$

$$A = m_1 + m_2 + m_3 = 1.51 + .523 + .345$$

$$B = m_1(m_2 + m_3) + m_2 m_3 = 1.51 + .868 + .523 \times .345 = 1.49$$

$$C = m_1 m_2 m_3 = 1.51 \times .523 \times .345 = .273$$

from dimensions of crystal

$l_y = 1.91$ cm

$l_w = 1.51$ cm

$l_t = 0.102$ cm

$$C_0 = .402 \frac{l_w l_y}{l_t} = \frac{.402 \times 1.51 \times 1.91}{.102} = 11.4 \ \mu\mu f$$

$$C_1 = 2.924 \times 10^{-3} \frac{l_w l_y}{l_t} = \frac{2.924 \times 1.51 \times 1.91 \times 10^{-3}}{.102} = .082 \ \mu\mu f$$

$$L_1 = 139 \frac{l_y l_t}{l_w} = \frac{139 \times 1.91 \times .102}{1.51} = 17.9 \text{ henry}$$

From equations in Table II

Computing Z_0

$$Z_0 = \frac{f_1(f_1^2 + Bf_3^2)}{2\pi C_1 f_3 (f_3 - f_1)(Af_1^2 + Cf_3^2)}$$

$$= \frac{4 \times 7.3 \times 3.25 \times 10^{13}}{.083 \times 10^{-12} \times 1.144 \times 4.57 \times 6.2 \times 10^{21}} = 353 \times 10^3 \ \Omega$$

Let $Z_0 = 350 \ K\Omega$

$$L_0 = \frac{Z_0(f_3 - f_1)(1 + B)}{2\pi f_1 f_3 (A + C)} = \frac{3.5 \times 4 \times 2.49 \times 10^5 \times 10^3}{1.144 \times 2.65 \times 10^{11}} = 11.5 \ mh$$

$$L_1 = \frac{Z_0(f_3 - f_1)(A + C)}{2\pi f_1 f_3 (1 + B)} = \frac{3.5 \times 10^5 \times 4 \times 10^3 \times 2.65}{1.144 \times 10^{11} \times 2.49} = 13 \ mh$$

$$C_0 = \frac{(Af_1^2 + Cf_3^2) f_3}{2\pi Z_0 f_1 (f_1 + f_3)(f_1^2 + Bf_3^2)} = \frac{4.71 \times 1.37 \times 10^{10} \times 10^5}{3.5 \times 2\pi \times 1.33 \times 4 \times 4.57 \times 10^{23}} = 121 \ \mu\mu f$$

$$C_1 = \frac{f_1(f_1^2 + Bf_3^2)}{2\pi Z_0 f_3(f_3 - f_1)(Af_1^2 + f_3^2)} = \frac{1.35 \times 4.57 \times 10^{15}}{3.5 \times 10^5 \times 2\pi \times 1.57 \times 4 \times 4.71 \times 10^{18}}$$

$$= 107\ \mu\mu f$$

Since split-plated crystals are used:

$L_0' = 2L_0 = 23\ mh$

$L_1' = 2L_1 = 26\ mh$

$C_0' = C_0/2 = 60.5\ mh$

$C_1' = C_1/2 = 53.5\ mh$

LOW IMPEDANCE FILTER DESIGN CALCULATIONS

Let $f_1 = 132.5$ kc $\qquad f_{\infty 1} = 130$ kc $\qquad f_{\infty 3} = \infty$

$f_3 = 137.5$ kc $\qquad f_{\infty 2} = 140$ kc

$$m_1 = \sqrt{\frac{1 - \overline{1.3}^2/\overline{1.375}^2}{1 - \overline{1.3}^2/\overline{1.325}^2}} = 1.71$$

$$m_2 = \sqrt{\frac{1 - \overline{1.4}^2/\overline{1.375}^2}{1 - \overline{1.3}^2/\overline{1.375}^2}} = .552$$

$$m_3 = \sqrt{\frac{f_1^2}{f_3^2}} = \frac{f_1}{f_3} = .964$$

$$A = m_1 + m_2 + m_3 = 3.226$$

$$B = m_1 m_2 + m_1 m_3 + m_2 m_3 = 3.122$$

$$C = m_1 m_2 m_3 = 1.71 \times .552 \times .964 = .91$$

$$Z_0 = \frac{C(AB - C)(f_3 - f_1)^3 (f_3 + f_1)^2}{2\pi C_3 f_3 f_1 \left[f_3^4 BC + 2 f_1^2 f_3^2 C + f_1^2 A(1+B) - C \right] \left[1 + B \right]}$$

$$= \frac{.91 \times 9.2 (5 \times 10^3)(2.7 \times 10^5)^2}{8.33 \times 2.7 \times 1.325 \times 1.375 \times 10^{-4} \left[\overline{1.375}^4 \times 3.12 \times .91 + 2 \times \overline{1.325}^2 \times \overline{1.375}^2 \times .91 + \overline{1.325}^4 (3.25 \times 4.12 - .91)(4.12 \times 10^{20}) \right)}$$

$$= 594\ \Omega$$

$$L_0 = \frac{Z_0 f_3 (f_1^2 A + f_3^2 C)}{2\pi f_1 (f_3 - f_1)(f_3^2 + f_1^2 B}$$

$$= \frac{594 \times 1.575 \times 10^5 (\overline{1.325}^2 \times 3.226 + \overline{1.375}^2 \times .91) 10^{10}}{2\pi \times 1.325 \times 10^5 \times 5 \times 10^3 (\overline{1.325}^2 + \overline{1.375}^2 \times 3.12) \times 10^{10}}$$

$$= 19.2 \text{ mh}$$

$$C_1 = \frac{(f_3 - f_1)(A f_1^2 + f_3^2 C)^2}{2\pi Z_0 f_1 f_3 [f_3^4 BC + 2 f_1^2 f_3^2 C + f_1^4 (A(1+B) - C]}$$

$$= \frac{5 \times 10^3 \times \overline{7.37}^2 \times 10^{20}}{2\pi \times 5.94 \times 10^2 \times 1.375 \times 10^{10} \times 3.27 \times 10^{21}}$$

$$= 122 \ \mu\mu f$$

since $m_3 = \dfrac{f_1}{f_3}$

$L_0 = L_1$ and $C_0 = C_1$

APPENDIX B

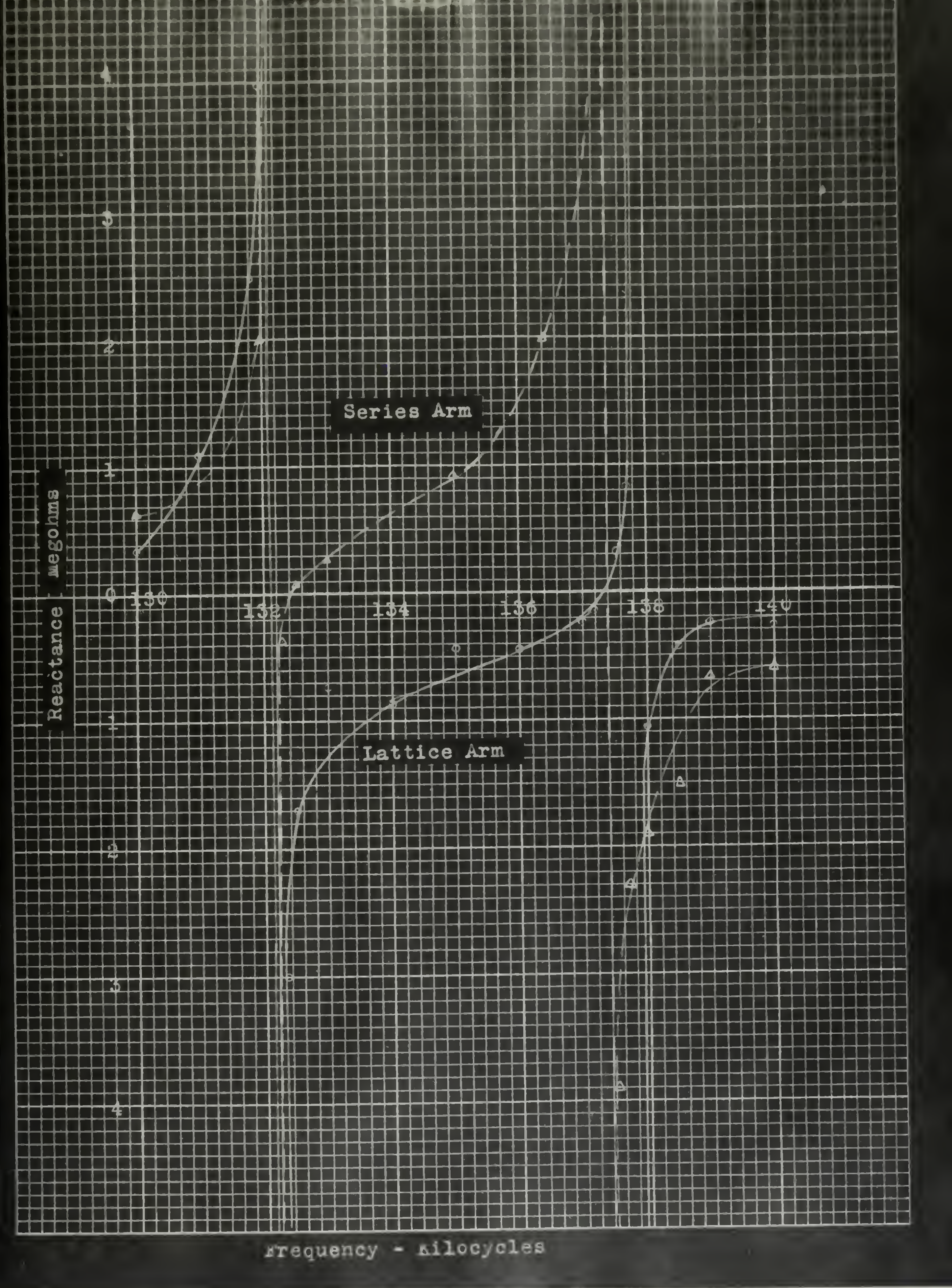
DATA FOR REACTANCE CURVES

	Series Arm				Lattice Arm		
Frequency (KC/S)	Meter C_1 ($\mu\mu f$)	Dial C_2 ($\mu\mu f$)	ΔC	Reactance (megohms)	C_2 ($\mu\mu f$)	ΔC	Reactance (megohms)
130	140.0	145.7	5.7	0.21	143.5	3.5	0.35
131	138.5	142.8	4.3	0.28	139.6	1.1	1.1
132	136.0	136.6	0.6	2.0	136.0	0	00
132.3	135.2	132	3.2	- 0.38	134.8	0.4	- 3.0
132.5	134.2	152.5	18.3	0.07	133.5	0.7	- 1.7
133.	133.3	137.7	4.4	0.27	131.7	1.6	- 0.74
135	131.2	129.9	1.3	0.91	125.7	2.9	- 0.41
137	128.6	124.9	0.2	5.8	119.1	5.6	- 0.21
137.2	126.9	124.1	0.3	- 4.1	115.3	9.1	- 0.13
137.5	124.7	123.8	0.3	- 3.9	126.	1.9	0.61
137.7	124.4	123.3	0.5	- 2.3	125.2	1.4	0.83
138.	123.4	122.8	0.6	- 1.9	122.3	1.1	- 1.05
138.5	121.8	121.0	0.8	- 1.4	119.2	2.8	- 0.41
139.	120.5	119.2	1.7	- 0.67	115.7	4.8	- 0.24
140	119.2	117.3	1.9	- 0.60	115.2	4.0	- 0.28

$$X = \frac{1.59\times10^{0}}{f\times\Delta C} \text{ ohms}$$

f - KCS

ΔC - $\mu\mu f$



Series Arm
Lattice Arm
Reactance megohms
130
132
134
136
138
140
Frequency - Kilocycles

ATTENUATION DATA

Low - Z Filter

Frequency KC	db Attenuation
130	31
131	36
131.5	52
132	5
132.4	4
132.5	5.2
133	1
133.5	0
134	1.3
135	2.7
136	0.3
137	0
137.5	2.2
138	3.6
139	28
139.5	50
140	34

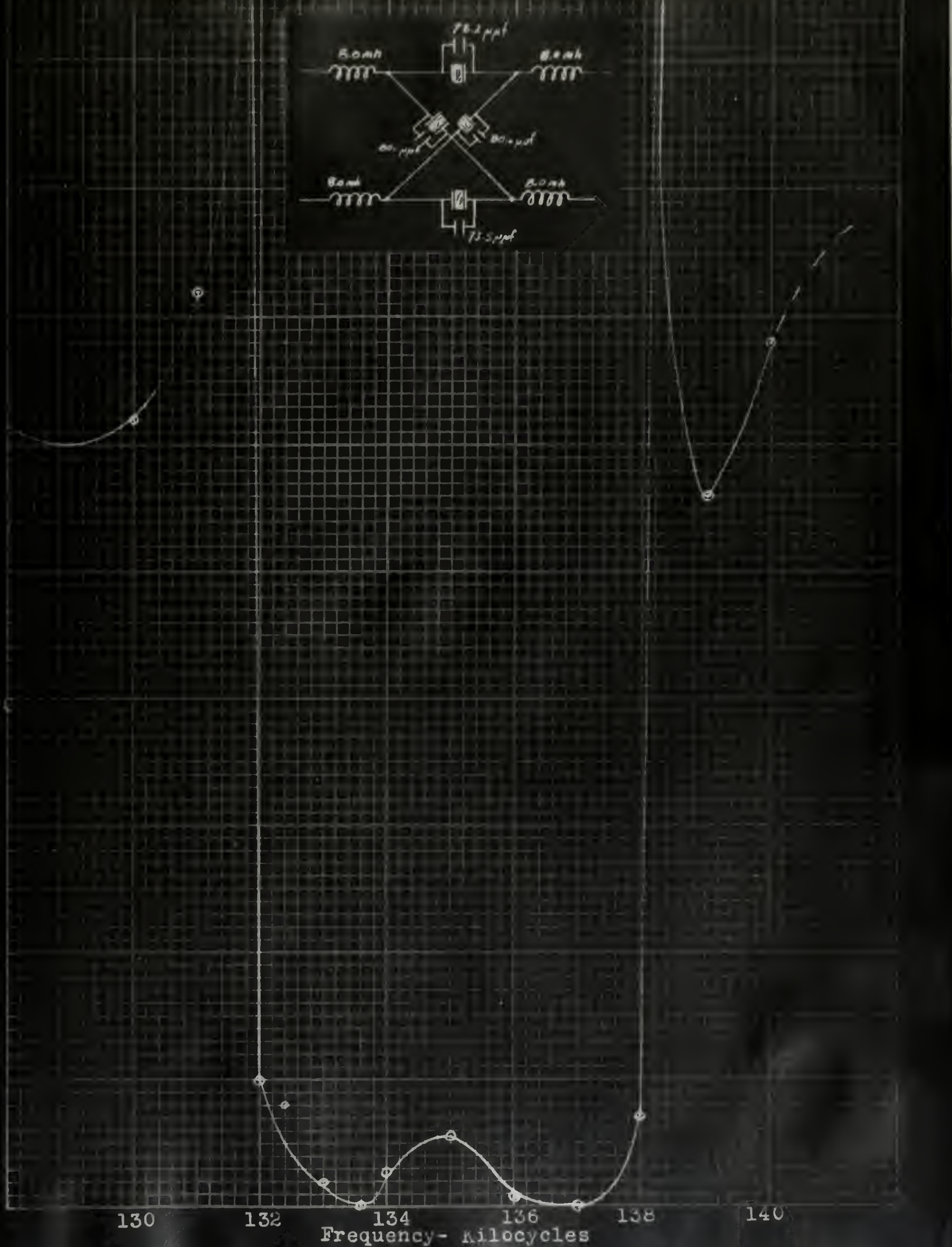
High Z Filter #1

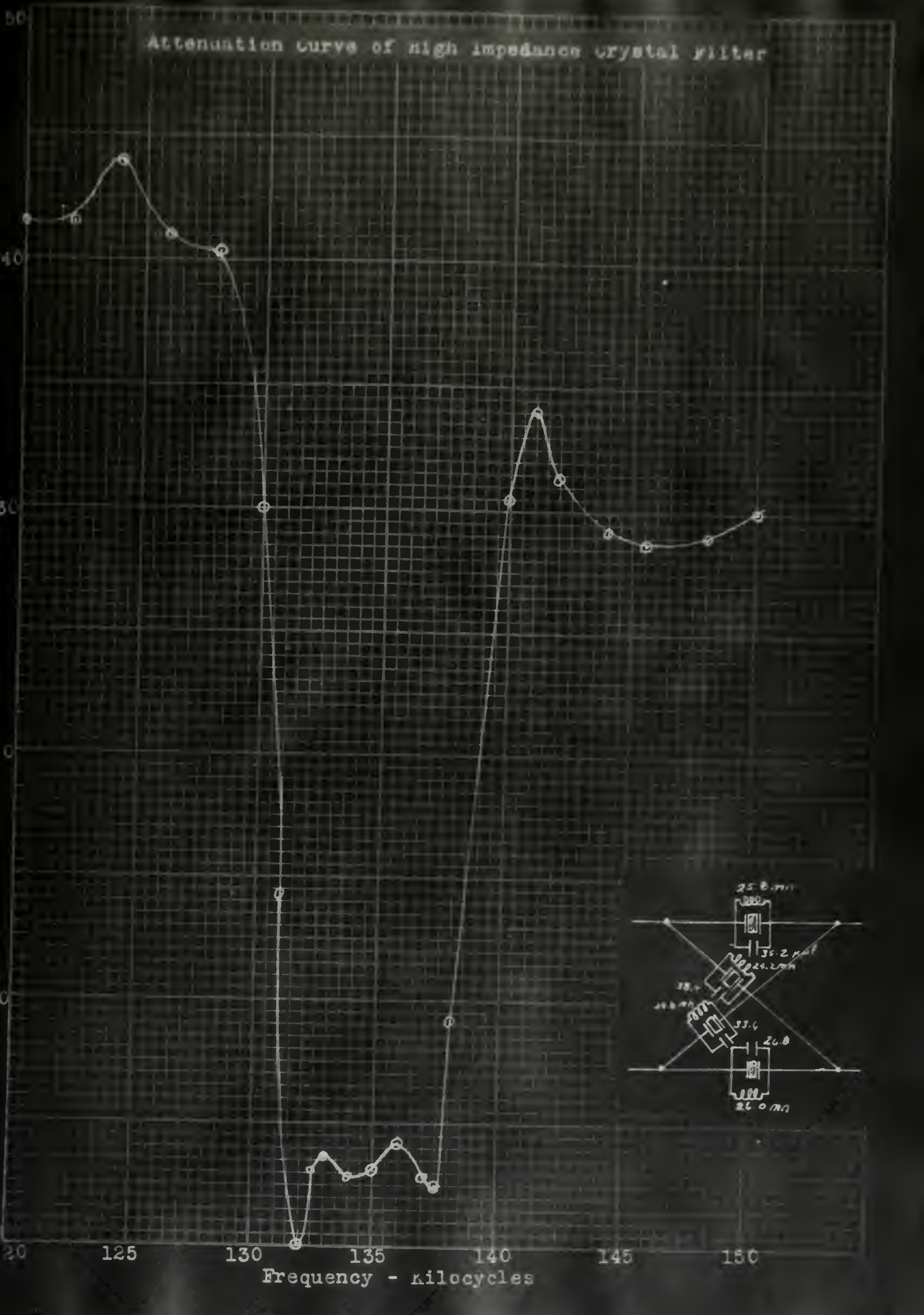
Frequency KC	db Attenuation
120	41.5
122	41.5
124	44
126	41
128	40.3
130	30
131	14.3
132	0
132.4	2.5
132.5	3.2
133	3.7
134	2.8
135	3.2
136	4.2
137	2.8
137.5	2.5
138	9.2
139	19.5
140	30.5
141	34
142	31.3
144	29.2
146	28.7
148	29
150	30

High Z Filter #2

Frequency KC	db Attenuation
120	28
122	23.9
124	20.4
126	20.2
128	20.2
130	22
131	47.9
132	0
132.5	1.2
133	7.6
134	2.0
135	1.2
136	2.9
137	3.2
137.5	3.2
138	8.6
139	27.2
139.5	59.6
141	30.4
142	27.
144	20.4
146	22
148	25
150	27

Attenuation Curve of Low Impedance Crystal Filter

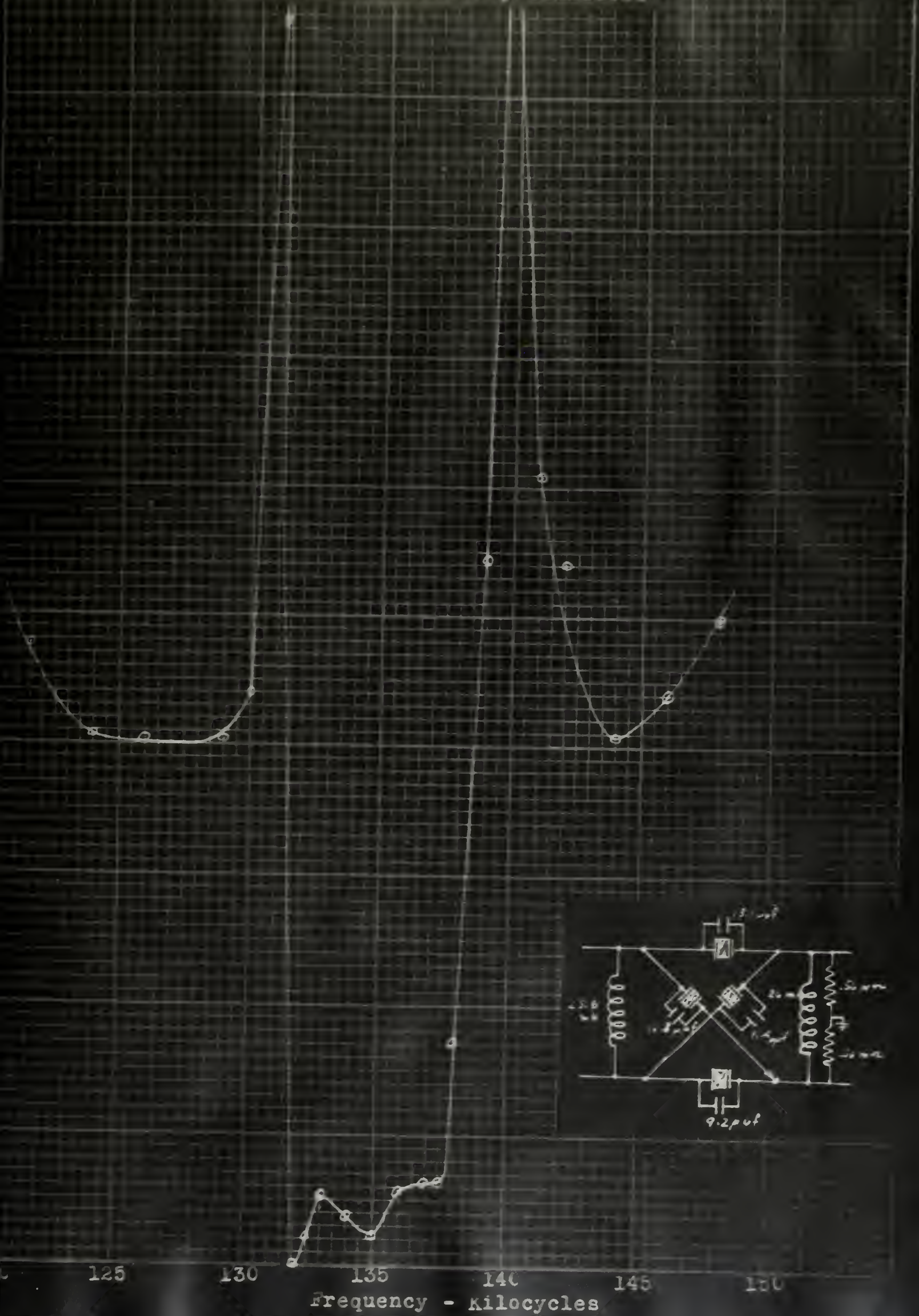




Attenuation Curve of High Impedance Crystal Filter
25.8 mh
35.2 mmf
24.2 mh
35.4
24.8 mh
33.6
26.8
26.0 mh
20
30
40
50
120
125
130
135
140
145
150
Frequency - Kilocycles

Attenuation Curve of High Impedance Crystal Filter

Shunt Coils Externally Connected



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